

# High-frequency Surface Waves in a Current Carrying Hot Plasma Column

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The propagation of high-frequency axially-symmetric surface waves along a current-carrying hot plasma column in a glass tube is investigated by means of a kinetic plasma model. The wave spectrum and the wave damping rate are obtained. Our results agree with the experimental data for surface wave propagation in a gas-discharge plasma at low pressures. It is found that the wave damping is mainly due to the Landau mechanism.

## 1. Introduction

Study of surface waves in bounded plasmas is of considerable interest in connection with plasma generation and diagnostics. In view of the fact that the geometry of most of the laboratory experiments is cylindrical, the propagation of surface waves along a plasma column actually takes place in reality. Accordingly, much research [1–10] has been carried out to investigate the dispersion properties of high-frequency surface waves in a cold cylindrical plasma considering numerous possibilities. In particular, the theoretical results were compared with the experimental data obtained for surface wave propagation in cylindrical gas-discharge plasmas [1, 4, 7, 9, 11–15]. This comparison shows that in the frequency domain  $\omega/\omega_{pe} \leq 0.4$  (or  $\omega \leq \omega_{pe}/(1 + \varepsilon)^{1/2}$ ), where  $\omega/2\pi$  is the wave frequency,  $\omega_{pe}$  electron plasma frequency and  $\varepsilon$  the glass permittivity, the wave dispersion of high-frequency axially-symmetric surface waves can be described exactly by the cold plasma approximation.

The surface wave propagation along a hot plasma column has been studied by using fluid equations [16, 17]. The influence of the electron drift on the wave spectrum is also examined [17]. In a few experimental works [17–21], the space damping rate (attenuation) of high-frequency surface waves was investigated accounting for various mechanisms

for the wave damping, e.g. collisional and Landau dampings, as well as the losses in the tube walls.

In this paper, we study theoretically the spectrum and the damping rate of the high-frequency surface waves in a current-carrying hot plasma column for a wide range of frequency spectrum (including also  $\omega > \omega_{pe}/(1 + \varepsilon)^{1/2}$ ). The latter are then compared with the available experimental data [17, 18]. The direct electron motion has been included in our theoretical model, so that we can have a better comparison with the experiments [17, 18] which note a drift of electrons. Since in the experiments with gas-discharge plasmas at low pressures the inequality  $\omega_{pe} R/c < 1$  ( $R$  is the radius of the plasma column,  $c$  is the speed of light) is usually satisfied, for  $k_z < \omega_{pe}/c$  ( $k_z$  is the axial wave number) we may treat the waves electrostatically [10]. Moreover, the electrostatic approximation allows us to obtain more exact analytic expressions convenient for comparison with the experimental data for the spectrum and the damping rate in those domains where the influence of electron thermal motion on wave dispersion is significant. The formulae obtained supplement the analysis of electrodynamic dispersion relation of axially-symmetric high-frequency surface waves considered in our previous work [22].

## 2. Wave Dispersion Relation

We examine the propagation of electrostatic axially-symmetric waves  $\propto \exp(-i\omega t + ik_z z)$  in a column (with a radius  $R$ ) of hot homogeneous non-isothermal current-carrying plasma bounded by dielectric with a permittivity  $\varepsilon$  and thickness

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$d = R_1 - R$  and by vacuum ( $\varepsilon = 1$  for  $r > R_1$ ). We assume that plasma's electrons and ions are driven in opposite directions along the  $z$ -axis of the cylinder. Because we investigate the propagation of surface waves only in the high-frequency domain, one can neglect the direct ion motion and assume that all the electrons move with the same direct velocity  $V$ . The dynamics of plasma particles and field is governed by linearized Vlassov and Poisson's equations:

$$\frac{\partial f_{1\alpha}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{1\alpha}}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha} \mathbf{E} \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{v}} = 0, \quad (1)$$

$$\nabla \cdot \mathbf{E} = 4\pi \varrho, \quad (2)$$

where  $f_\alpha = f_{0\alpha} + f_{1\alpha}$  ( $f_{1\alpha} \ll f_{0\alpha}$ ) are the distribution functions of particles of species  $\alpha$  ( $\alpha = e, i$ ),  $e_\alpha$  and  $m_\alpha$  are the charges and the masses,  $\varrho$  is the charge number density into the plasma, and  $\mathbf{E} = -\nabla\varphi$  is the electric field. We solve Eqs. (1), (2) in a cylindrical frame of reference  $(r, \theta, z)$  assuming mirror reflection of the plasma particles at plasma-dielectric interface:

$$f_\alpha(r = R, -v_r) = f_\alpha(r = R, v_r) \quad (3)$$

and presuming Doppler-shifted equilibrium Maxwellian distributions:

$$f_{0\alpha}(v) = n_\alpha \left( \frac{m_\alpha}{2\pi T_\alpha} \right) \cdot \exp \left\{ -\frac{m_\alpha}{2T_\alpha} [v_r^2 + (v_z - V_\alpha)^2] \right\} \quad (4)$$

with  $|\mathbf{v}| = [v_r^2 + (v_z - V_\alpha)^2]^{1/2}$  and  $V_i = 0$ . We shall seek the solutions of (1) and (2) under boundary condition (3) in the form of plane waves:

$$\begin{aligned} \Psi_\alpha^-(r) &= \sum_{n=-\infty}^{\infty} \Psi_{\alpha n}^- \sin \kappa_n r, \\ \Psi_\alpha^+(r) &= \sum_{n=-\infty}^{\infty} \Psi_{\alpha n}^+ \cos \kappa_n r, \\ \varphi(r) &= \sum_{n=-\infty}^{\infty} \Phi_n \cos \kappa_n r, \end{aligned} \quad (5)$$

where

$$\begin{aligned} \kappa_n &= n\pi/R, \\ \Psi_\alpha^\mp(r, v_r) &= f_{1\alpha}(r, v_r) \mp f_{1\alpha}(r, -v_r). \end{aligned}$$

By defining the functions  $\Psi_\alpha^\mp$ , the boundary condition (3) becomes

$$\Psi_\alpha^-(r = R, v_r) = 0. \quad (6)$$

Moreover, we also have  $\Psi_\alpha^-(r=0, v_r) = 0$  — a kinematic condition which means that the radial component of the current density vanishes at the cylinder axis.

From Eq. (1) we obtain for the coefficients  $\Psi_{\alpha n}^\mp$  the following expressions:

$$\begin{aligned} \Psi_{\alpha n}^- &= -2ie_\alpha \frac{\omega - k_z V_\alpha}{v_r} \frac{\kappa_n}{\beta^2 - \kappa_n^2} f'_{0\alpha} \Phi_n, \\ \Psi_{\alpha n}^+ &= -2e_\alpha \frac{1}{\beta^2 - \kappa_n^2} \cdot \left[ \kappa_n^2 + \beta \frac{k_z(v_z - V_\alpha)}{v_r} \right] f'_{0\alpha} \Phi_n, \end{aligned} \quad (7)$$

where  $\beta = (\omega - k_z v_z)/v_r$  and  $f'_{0\alpha}$  is the derivative of the equilibrium distribution function with respect to the energy.

The components of the current density

$$j_{r,z}(r) = \sum_\alpha e_\alpha \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dv_r \Psi_\alpha^\mp(r) v_{r,z} \quad (8)$$

and the charge density

$$\varrho(r) = \sum_\alpha e_\alpha \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dv_r \Psi_\alpha^+(r), \quad (9)$$

are then found to be

$$\begin{aligned} j_r(r) &= \sum_{n=-\infty}^{\infty} \left\{ \sum_\alpha \left( -i \frac{e_\alpha^2 n_\alpha}{m_\alpha} \right) \frac{\omega - k_z V_\alpha}{k_n^2 v_{T\alpha}^2} \kappa_n \cdot [1 - J_+(x_\alpha)] \Phi_n \right\} \sin \kappa_n r, \\ j_z(r) &= \sum_{n=-\infty}^{\infty} \left\{ \sum_\alpha \left( -\frac{e_\alpha^2 n_\alpha}{m_\alpha} \right) \frac{k_z \omega + \kappa_n^2 V_\alpha}{k_n^2 v_{T\alpha}^2} \cdot [1 - J_+(x_\alpha)] \Phi_n \right\} \cos \kappa_n r, \\ \varrho(r) &= \sum_{n=-\infty}^{\infty} \left\{ \sum_\alpha \left( -\frac{1}{4\pi} \right) \cdot [\varepsilon^L(\omega, k_n) - 1] k_n^2 \Phi_n \right\} \cos \kappa_n r, \end{aligned} \quad (10)$$

when  $n_\alpha$  is particles' number density,  $v_{T\alpha} = (T_\alpha/m_\alpha)^{1/2}$  are the thermal velocities,  $x_\alpha = (\omega - k_z V_\alpha)/|\mathbf{k}_n| v_{T\alpha}$ ,  $k_n^2 = \kappa_n^2 + k_z^2$ ,

$$J_+(x_\alpha) = x_\alpha \exp(-x_\alpha^2/2) \int_{+i\infty}^{x_\alpha} dy \exp(y^2/2),$$

and

$$\varepsilon^L(\omega, k_n) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{k_n^2 v_{T\alpha}^2} [1 - J_+(x_\alpha)]$$

is the longitudinal dielectric function of an infinite homogeneous isotropic Maxwellian plasma [23]. Here  $\omega_{p\alpha}^2 = (4\pi n_\alpha e_\alpha^2)/m_\alpha$ .

On equating the expressions for  $\varrho(r)$  defined by (9) and that found from the equation of continuity

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot \mathbf{j} = 0,$$

one obtains

$$\sum_{n=-\infty}^{\infty} \frac{1}{r} \left\{ (\varepsilon^L - 1) - \frac{k_z V}{\omega} \frac{\omega_{pe}^2}{k_n^2 v_{Te}^2} \right. \\ \left. \cdot [1 - J_+(x_e)] \right\} \kappa_n \Phi_n \sin \kappa_n r = 0. \quad (11)$$

From Poisson's Eq. (2) we find

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - k_z^2 \right) \eta(r) = \frac{1}{r} \frac{d}{dr} \sum_{n=-\infty}^{\infty} \frac{k_z V}{\omega} \\ \frac{\omega_{pe}^2}{k_n^2 v_{Te}^2} [1 - J_+(x_e)] \Phi_n \cos \kappa_n r, \quad (12)$$

where

$$\eta(r) = \sum_{n=-\infty}^{\infty} \varepsilon^L \Phi_n \cos \kappa_n r,$$

and Eq. (11) has been used.

For

$$\omega < \omega_{pe} \quad \text{and} \quad |k_z V| \ll \omega, \quad (13)$$

the term on the right-hand side of (12) can be neglected, and one finds

$$\eta(r) = C I_0(k_z r)$$

or [see (5)]:

$$\varphi_{\text{plasma}}(r) = \frac{C}{R} \sum_{n=-\infty}^{\infty} \frac{1}{\varepsilon^L(\omega, k_n)} \\ \cdot \cos \kappa_n r \int_0^R dr' I_0(k_z r') \cos \kappa_n r', \quad (14)$$

where

$$\Phi_n = \frac{1}{\varepsilon^L(\omega, k_n)} \frac{C}{R} \int_0^R dr I_0(k_z r) \cos \kappa_n r$$

( $C$  being a constant). Having derived the coefficients  $\Phi_n$  ( $\eta(r)$  respectively), on the basis of equality

$$D_r(r) = -\frac{\partial \varphi(r)}{\partial r} + \frac{4\pi i}{\omega} j_r(r)$$

and taking into account conditions (13), for the radial component of the electric displacement vector we get

$$D_r(r) = -C k_z I_1(k_z r). \quad (15)$$

In (14) and (15)  $I_0$  and  $I_1$  are the modified Bessel functions of the first kind.

The boundary conditions for the continuity of the potential  $\varphi$  and the radial component of the electric displacement vector  $D_r$  at the plasma-dielectric and dielectric-vacuum interfaces yield the following dispersion relation for axially-symmetric waves in a current-carrying hot plasma column:

$$\Delta(\omega, k_z) \equiv 1 - G_0 \sum_{n=-\infty}^{\infty} (-1)^n \frac{A_n}{\varepsilon^L(\omega, k_n)} = 0, \quad (16)$$

where the geometrical factor  $G_0$  has the form:

$$G_0 = \varepsilon \frac{(\varepsilon - 1) I_1(a) K_0(b) K_1(b) - K_1(a) [I_0(b) K_1(b) + \varepsilon I_1(b) K_0(b)]}{(\varepsilon - 1) I_0(a) K_0(b) K_1(b) + K_0(a) [I_0(b) K_1(b) + \varepsilon I_1(b) K_0(b)]} \quad (17)$$

(with  $a = k_z R$  and  $b = k_z R_1$ ;  $K_0$  and  $K_1$  being the modified Bessel functions of the second kind). The expression

$$A_n = \frac{1}{R} \frac{1}{I_1(k_z R)} \int_0^R dr I_0(k_z r) \cos \kappa_n r \quad (18)$$

can be calculated for thick ( $k_z R > 1$ ) and thin ( $k_z R < 1$ ) cylinders:

$$\frac{(-1)^n}{\kappa_n^2 + k_z^2} \frac{k_z}{R} \left[ 1 + \frac{k_z}{R} \frac{1}{\kappa_n^2 + k_z^2} \frac{I_0(k_z R)}{I_1(k_z R)} \right] \quad \text{at} \quad k_z R > 1 \quad (19)$$

$$A_n = \left\{ \begin{array}{ll} \frac{(-1)^2}{\kappa_n^2 + k_z^2} \frac{k_z}{R} \left( 1 - \frac{3}{4} \frac{k_z^2}{\kappa_n^2 + k_z^2} \right) & \text{when } n \neq 0 \\ \frac{2}{k_z R} \left[ 1 - \frac{(k_z R)^2}{24} \right] & \text{when } n = 0 \end{array} \right\} \quad \text{at} \quad k_z R < 1. \quad (20)$$

When  $V=0$ ,  $R_1=R$ , from (16) we get the electrostatic approximation of the exact electrodynamic dispersion relation obtained in Reference [22]. On the other hand, for  $v_{T\alpha}=0$ ,  $R_1=R$  one finds the electrostatic limit of surface wave dispersion relation in a current-carrying cold plasma derived in Ref. [10] by using fluid equations.

### 3. Analysis of the Dispersion Relation and Comparison with the Experiments

Let us split  $\varepsilon^L(\omega, k_n)$  and  $\Delta(\omega, k_z)$  into their real and imaginary parts. Then from equation

$$\operatorname{Re} \Delta(\omega, k_z) \equiv 1 - G_0 \sum_{n=-\infty}^{\infty} (-1)^n \cdot \frac{\operatorname{Re} \varepsilon^L(\omega, k_n)}{|\varepsilon^L(\omega, k_n)|^2} A_n = 0 \quad (21)$$

we obtain the wave spectrum and the damping rate

$$\gamma_\omega = - \frac{\operatorname{Im} \Delta(\omega, k_z)}{\frac{\partial}{\partial \omega} \operatorname{Re} \Delta(\omega, k_z)}, \quad (22)$$

where

$$\operatorname{Im} \Delta(\omega, k_z) \equiv G_0 \sum_{n=-\infty}^{\infty} (-1)^n \cdot \frac{\operatorname{Im} \varepsilon^L(\omega, k_n)}{|\varepsilon^L(\omega, k_n)|^2} A_n. \quad (23)$$

We look for high-frequency ( $\omega^2 \gg \omega_{pi}^2$ ) surface waves in long-wavelength approximation, viz.

$$\frac{\omega - k_z V}{k_z v_{Te}} \gg 1, \quad \frac{\omega}{k_z v_{Ti}} \gg 1 \quad (|k_z V| < \omega). \quad (24)$$

Accordingly, the wave frequency can be written as

$$\omega = \omega_0 + \omega_1, \quad (25)$$

where  $\omega_0$  is the frequency of the surface wave obtained from equation

$$\Delta_0(\omega, k_z) \equiv 1 - G_0 \sum_{n=-\infty}^{\infty} (-1)^n \frac{A_n}{\varepsilon(\omega)} = 0 \quad (26)$$

with  $\varepsilon(\omega) = 1 - \omega_{pe}^2/(\omega - k_z V)^2$ .  $\omega_1$  is a correction to the spectrum of the waves in a cold plasma. This contribution arises owing to the thermal motion of the plasma particles and is given by

$$\omega_1 = - \frac{\Delta_1(\omega_0, k_z)}{\frac{\partial}{\partial \omega} \Delta_0(\omega, k_z)|_{\omega=\omega_0}}, \quad (27)$$

where

$$\Delta_1(\omega, k_z) \equiv -G_0 \sum_{n=-\infty}^{\infty} (-1)^n \cdot \left[ \frac{1}{\operatorname{Re} \varepsilon^L(\omega, k_n)} - \frac{1}{\varepsilon(\omega)} \right] A_n. \quad (28)$$

From (26) we obtain the spectrum of high-frequency surface waves propagating along a current-carrying cold plasma column bounded by a dielectric tube immersed in vacuum:

$$\omega_0 = \frac{\omega_{pe}}{\sqrt{1+B_0}} \pm |k_z V|, \quad (29)$$

where

$$B_0 = -G_0 [I_0(k_z R)/I_1(k_z R)].$$

The latter expression was derived by means of the identity

$$I_0(k_z R)/I_1(k_z R) = \sum_{n=-\infty}^{\infty} (-1)^n A_n \quad (30)$$

and is valid for arbitrary values of  $k_z R$ .

In obtaining both the thermal correction  $\omega_1$  to the spectrum of the waves and the wave damping rate  $\gamma_\omega$  we analyze formulae (27) and (22) for thick ( $k_z R > 1$ ) and thin ( $k_z R < 1$ ) cylinders, by assuming  $|\operatorname{Re} \varepsilon^L(\omega, k_n)|^2 \gg |\operatorname{Im} \varepsilon^L(\omega, k_n)|^2$ . Further, in calculation we will use only the asymptotic values of the function  $J_+(x_\alpha)$  [23] (in the expression for  $\varepsilon^L(\omega, k_n)$ ) for small and large values of its argument  $x_\alpha$ . Let

$$\kappa_{n1} = k_z ((\omega - k_z V)^2 / k_z^2 v_{Te}^2 - 1)^{1/2}$$

be that value of the radial wave number which corresponds to

$$x_e = (\omega - k_z V) / |k_n| v_{Te} \sim 1,$$

and

$$\kappa_{n2} = k_z (\omega^2 / k_z^2 v_{Ti}^2 - 1)^{1/2}$$

be the value of  $\kappa_n$  which corresponds to

$$x_i = \omega / |k_n| v_{Ti} \sim 1.$$

Following the manner of calculation of the analogous problem for semi-infinite plasma [24], we divide the region of summation over  $n$  in the expressions for  $\Delta_1(\omega, k_z)$  and  $\operatorname{Im} \Delta(\omega, k_z)$  into the following intervals:

i)  $\kappa_n \in (0, \kappa_{n1})$ . In this region  $x_\alpha \gg 1$  and one may use the asymptotic expansion of function  $J_+(x_\alpha)$  for large values of its argument  $x_\alpha$ .

ii)  $\kappa_n \in (\kappa_{n1}, \kappa_{n2})$ . In summation one should note that here  $x_e \ll 1$ ,  $x_i \gg 1$  and the corresponding expansions of  $J_+(x_\alpha)$  for small and large values of the argument  $x_\alpha$  should be used.

iii)  $\kappa_n \in (\kappa_{n2}, \infty)$ , where  $x_\alpha \ll 1$  and, therefore, one uses the asymptote of  $J_+(x_\alpha)$  for small values of  $x_\alpha$ .

Figure 1 shows schematically the dependence among the variables  $\kappa_n$ ,  $x_e$  and  $x_i$ .

For a thick cylinder,  $k_z R > 1$ , from (28) [with  $A_n$  given by (19)] we find

$$A_1(\omega, k_z) = -G_0 k_z R \sum_{n=-\infty}^{\infty} g(z), \quad (31)$$

$$\begin{aligned} g(z) &= \frac{l^2}{q - k_z^2 l^2} \frac{1}{l^2 z^2 + q R^2} \left( 1 + \frac{a}{z^2 + a^2} \frac{I_0(a)}{I_1(a)} \right), \\ g(z) &= \left( \frac{1}{z^2 + a^2} \frac{1}{B_0} + \frac{1}{z^2 + \left( k_z^2 + \frac{1}{r_{De}^2} \right) R^2} \right) \left( 1 + \frac{a}{z^2 + a^2} \frac{I_0(a)}{I_1(a)} \right), \\ g(z) &= \left( \frac{1}{z^2 + a^2} \frac{1}{B_0} + \frac{1}{z^2 + \left( k_z^2 + \frac{1}{r_{Di}^2} \right) R^2} \right) \left( 1 + \frac{a}{z^2 + a^2} \frac{I_0(a)}{I_1(a)} \right) \end{aligned} \quad (32)$$

with

$$l^2 = 3\omega_{pe}^2 v_{Te}^2 / (\omega - k_z V)^4, \quad q = \omega_{pe}^2 / (\omega - k_z V)^2 - 1 + k_z^2 l^2 \quad \text{and} \quad r_{D\alpha} = (T_\alpha / 4\pi n_\alpha e_\alpha^2)^{1/2}$$

being the Debye length. The summation over  $n$  in (31) is taken by using the Cauchy residue theorem. Finally, we obtain (neglecting the terms of the order  $(T_i/T_e)^{1/2}$ ) the following spectrum for the high-frequency surface waves propagating along a thick ( $k_z R > 1$ ) plasma column:

$$\omega = \frac{\omega_{pe}}{\sqrt{1+B_0}} \left\{ 1 + \frac{1}{2} |G_0| \left[ \sqrt{\frac{3}{B_0}} + \frac{B_0}{1+B_0} + 3 \frac{r_{De}}{R} \frac{1+B_0}{p} \right] k_z r_{De} \right\} \pm |k_z V|, \quad (33)$$

where  $p = |G_0|$ . The values of geometrical factors  $G_0$  and  $B_0$  are given by the expressions:

$$\begin{aligned} G_0 &= \varepsilon \frac{(\varepsilon - 1) \{ (1 - 3/8a + 1/4b) \exp(-2k_z d) + 1/2b \} - (1 + \varepsilon)(1 + 3/8a)}{(\varepsilon - 1) \{ (1 + 1/8a + 1/4b) \exp(-2k_z d) - 1/2b \} + (1 + \varepsilon)(1 - 1/8a)}, \\ B_0 &= -G_0(1 + 1/2a). \end{aligned} \quad (34)$$

It is of interest to mention that depending on the thickness of the dielectric  $d = R_1 - R$ , the values of  $|G_0|$  and  $B_0$  lie in the interval  $(1, \varepsilon)$  [17].

For a thin cylinder,  $k_z R < 1$ , as  $(v_{Te}/v_{ph}) < k_z R$  ( $v_{ph}$  is the wave phase velocity) by using expression (20) for  $A_n$  we obtain the same spectrum (33), but now  $p = B_0$  and  $|G_0|$  and  $B_0$  are given by [17]:

$$\begin{aligned} |G_0| &\cong \frac{\varepsilon}{k_z R} \frac{1}{\varepsilon \ln(1/k_z R_1) + \ln(R_1/R)}, \\ B_0 &= \frac{2}{k_z R} |G_0|. \end{aligned} \quad (35)$$

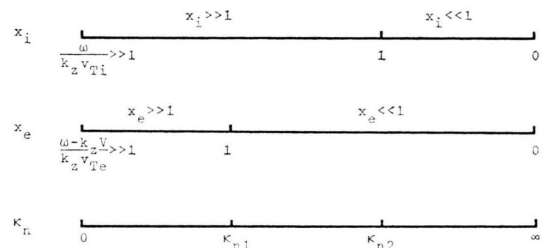


Fig. 1. Relations among variables  $x_i$ ,  $x_e$  and  $\kappa_n$ .

where  $z = n\pi$  and the function  $g(z)$  has the following form in the three intervals i), ii) and iii) discussed above:

When  $(v_{Te}/v_{ph}) > k_z R$ , we obtain the spectrum

$$\begin{aligned} \omega &= \frac{\omega_{pe}}{\sqrt{1+B_0}} \left[ 1 + \frac{1}{2} |G_0| k_z r_{De} \right. \\ &\quad \cdot \left. \left( 1 + \frac{21}{4} \frac{r_{De}}{R} \right) \right] \pm |k_z V| \end{aligned} \quad (36)$$

where  $|G_0|$  and  $B_0$  are given by (35).

We note that the electrostatic approximation in the limit of thin cylinder is valid if

$$\omega_{pe} R/c < k_z R < 1.$$

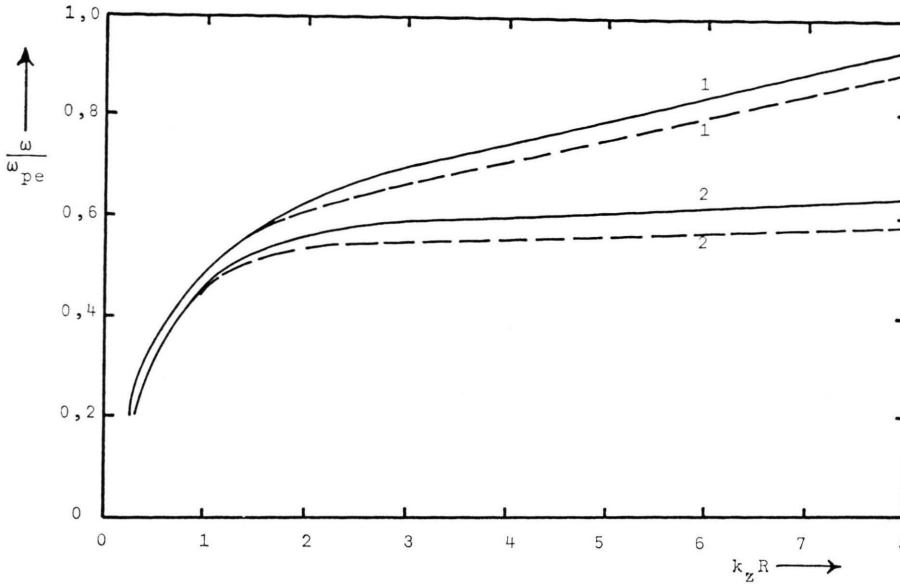


Fig. 2. Dispersion curves for axially-symmetric high-frequency surface waves propagating along a current-carrying hot plasma column ( $\omega_{pe}/2\pi = 230$  MHz,  $v_{Te} = 1.1 \cdot 10^8$  cm/s,  $V = 5.8 \cdot 10^7$  cm/s,  $R = 2.10$  cm,  $R_1 = 2.25$  cm,  $\varepsilon = 4.4$ ); — kinetic description; --- hydrodynamic description; 1 downstream waves; 2 upstream waves.

Therefore, formula (33) obtained at  $(v_{Te}/v_{ph}) < k_z R$  gives the spectrum of those surface waves for which  $(v_{ph}/c) > (r_{De}/R)$ , while (36) found at  $(v_{Te}/v_{ph}) > k_z R$  corresponds to the waves with  $(v_{ph}/c) < (r_{De}/R)$ . Since the concept of plasma makes sense only if  $(r_{De}/R) \ll 1$  and because the wave phase velocity  $v_{ph}$  increases with the decrease of the frequency, obviously only the first case,  $(v_{Te}/v_{ph}) < k_z R$ , describes an actual situation.

The influence of the electron drift on the spectrum of high-frequency surface waves manifests in a Doppler displacement of the wave frequency with respect to the frequency in a plasma without current. This leads to an increase of the wavelengths of the waves propagating in the direction of the current (downstream waves) and to a decrease of the wavelengths of the waves running in opposite direction (upstream waves). Formula (33) obtained includes also the influence of the thermal electron motion on the wave spectrum. The motion is taken into

account by means of the kinetic plasma model. The comparison of this formula with the corresponding expression in Ref. [17], obtained on the basis of a fluid plasma model, shows that the kinetic thermal corrections to the spectrum of a cold plasma (29) are larger than those found from fluid equations. Figure 2 illustrates this difference.

To calculate the wave damping rate  $\gamma_\omega$ , more precisely, the expression for  $\text{Im } \Delta(\omega, k_z)$  (23), we replace the summation with integration using the asymptotic formula [22]:

$$\lim_{\delta \rightarrow +0} \delta \sum_{n=0}^{\infty} f(n\delta) = \int_0^{\infty} dx f(x),$$

where  $\delta \equiv \pi v_{Te}/R(\omega - k_z V)$ . In calculations we accounted for the contributions of  $J_+(x_\alpha)$  into  $\text{Im } \Delta(\omega, k_z)$  in the intervals i), ii) and iii) (Fig. 1) as in the first region  $|\text{Re } \varepsilon^L(\omega, k_n)|^2$  is replaced by its value at the point of maximum wave damping ( $x_e \sim 1$ ). Finally, we find assuming  $(v_{Te}/v_{ph}) < k_z R$ :

$$\gamma_\omega = -\frac{\omega_{pe}}{\sqrt{1+B_0}} \frac{|G_0| B_0}{\sqrt{2\pi}(1+B_0)} k_z r_{De} \left\{ \frac{3}{\sqrt{e}} \frac{(1+B_0)^{3/2}}{(3+4B_0)^2} + \frac{1}{\sqrt{1+B_0}} \left( \ln \sqrt{2+B_0} - \frac{1}{2} \frac{1+B_0}{2+B_0} \right) + s k_z^2 r_{De}^2 (1+B_0) \left[ \frac{13}{\sqrt{e}} \frac{(1+B_0)^{3/2}}{(3+4B_0)^2} + \frac{1}{\sqrt{1+B_0}} \left( \frac{1}{2} \frac{3+B_0}{2+B_0} - \frac{2}{1+B_0} \ln \sqrt{2+B_0} \right) \right] \right\}, \quad (37)$$

where

$$s = \frac{1}{k_z R} \frac{I_0(k_z R)}{I_1(k_z R)}$$

when  $k_z R > 1$  and  $s = -3/4$  for  $k_z R < 1$ . In the case of thick column, the values of  $|G_0|$  and  $B_0$  in

(37) are given by (34), whereas for thin column by (35). Furthermore, the value of  $\gamma_\omega$  in the limit  $k_z R < 1$  is true if

$$\omega > \omega_{pe} \left( \frac{T_i}{T_e} \frac{m_e}{m_i} \right)^{1/2}.$$



From results obtained for spectrum (33) and damping rate (37) one may conclude that the influence of electron thermal motion on the dispersion of the waves is stronger in the case of thick column. The dependence of geometrical factors  $|G_0|$  and  $B_0$  on  $k_z R$  provides a monotonous variation of the wave spectrum and the damping rate with the change of the wavelength and the radius of the plasma column.

The wave attenuation, requisite for comparison of the theoretical results with the experimental data, can be found by the formula  $\gamma_k = -\gamma_\omega/v_{gr}$ , where the group velocity  $v_{gr}$  is determined by a graphic differentiation of the dispersion curve  $\omega(k_z)$  drawn according to formula (33). As a result of the difference in the group velocities of the waves propagating along the current and in opposite direction, one concludes that the downstream waves are more weakly damped than the upstream ones.

Figures 3 and 4 show the theoretical results obtained here and the experimental data for propagation of axially-symmetric high-frequency surface waves along a plasma column [17]. The theory agrees quite well with the experiment. The difference in terms accounting for the thermal

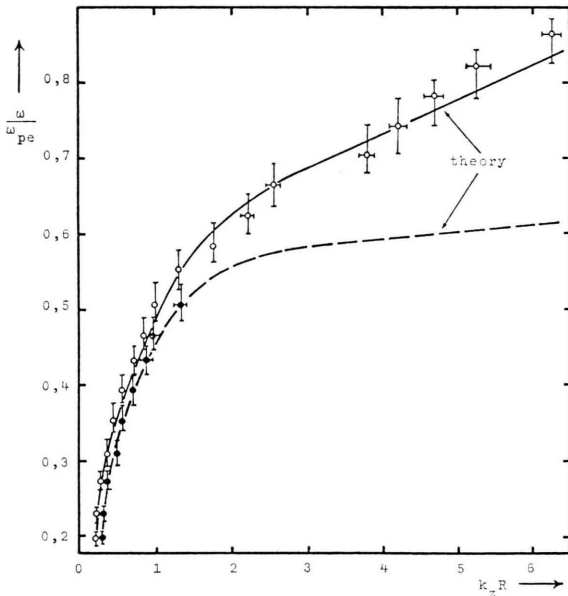


Fig. 3. Theoretical curves and experimental data for the spectrum of axially-symmetric downstream (—○) and upstream (---●) surface waves propagating in a Neon gas-discharge plasma [17] (pressure  $3 \cdot 10^{-3}$  Torr, discharge current 120 mA,  $R = 2.10$  cm,  $R_1 = 2.25$  cm,  $\epsilon = 4.4$ ,  $\omega_{pe}/2\pi = 256$  MHz,  $v_{Te} = 1.2 \cdot 10^8$  cm/s and  $V = 5.5 \cdot 10^7$  cm/s).

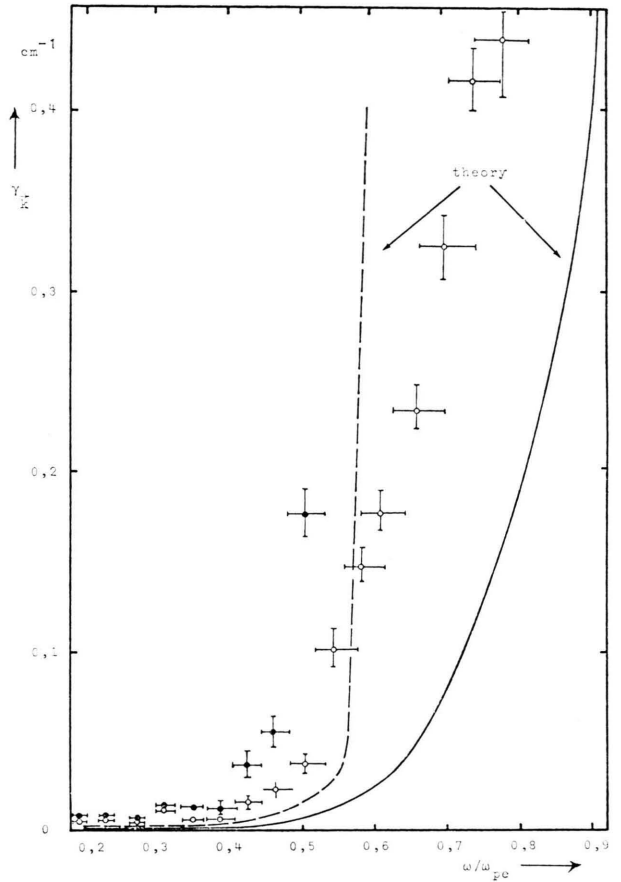


Fig. 4. The wave attenuation according to our theory and the experiment [17] (the same conditions as in Fig. 3); —○ downstream waves; ---● upstream waves.

electron motion on the basis of a fluid and a kinetic plasma models are within the limits of the experimental errors in determining the wavelengths and the plasma parameters. From Fig. 4 one may see that the damping of the surface waves in a gas-discharge plasma at low pressures is due mostly to the energy exchange between the wave and plasma electrons, i.e. to the Landau damping mechanism.

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